

Problem Set #2

(due 11/24/15)

1. Consider an economy with two commodities, X and Y , each produced competitively using a single factor, labor, which is in fixed overall supply, $L = L_x + L_y$. Producers of X have the production function $X = (\alpha e^{-\beta Y})L_x$ and producers of Y have the production function $Y = \gamma L_y$. That is, each producer perceives constant returns to scale with respect to labor, but labor productivity in sector X faces a negative externality based on the aggregate production of Y .
 - A. Derive the economy's production possibilities frontier as an expression for X in terms of Y , and show that the production set is not convex.
 - B. Letting labor be numeraire, derive the cost functions for producers of each good, $c(X;Y)$ and $c(Y)$, assuming that producers of X take Y as given and that producers of Y ignore their impact on sector X . Solve for the competitive prices at given values of X and Y .
 - C. Now, solve for the social cost function for X and Y , $c(X,Y)$, and the marginal social costs of X and Y at given production levels. Derive expressions for Pigouvian taxes on producers of X and Y that would cause competitive prices to equal marginal social costs.
 - D. Suppose that consumers are identical, with preferences that satisfy the utility function $U(X,Y) = X+Y$. How much revenue does the Pigouvian tax raise at the social optimum?
2. Consider an economy with overlapping generations, each with a single agent who lives for two periods. The interest rate is fixed at r . Government debt is issued at the beginning of the period and taxes, transfer payments and government purchases occur at the end of the period. Initially, the government has no debt outstanding and operates a social security system that transfers 1 dollar to the older individual from the younger individual in each period.
 - A. Write down the government's intertemporal budget constraint (GIBC) as of the beginning of year t , in terms of initial national debt, government purchases and government net taxes (taxes less transfers); show that the government's policy satisfies the GIBC.
 - B. Now write down the GIBC in its alternative formulation, in terms of the initial level of debt, government purchases, and the generational accounts for the two existing and all future generations. (Use the convention that generation t is the generation that is young in year t). Solve for the generational account for each generation, and show that this version of the GIBC is also satisfied under current government policy.
 - C. Suppose that, at the beginning of period $t+1$, the government (1) ends social security for all but current retirees, whose benefits it pays by issuing debt; and (2) meets debt service going forward using a constant tax on each subsequent elderly generation. Show that all generational accounts are the same as under the original social security system in part B.
 - D. Now, compare the generational accounts as of date $t+1$ for the initial social security system and the post-reform system. Do the accounts differ? Is the GIBC still satisfied?

3. Consider an individual who wishes to invest initial wealth, W , to maximize the utility of terminal wealth one period hence. The investor's problem consists of two decisions:
 - (1) how much of this wealth to place in bonds, which yield a certain return, $i > 0$, and how much to invest in stocks, which yield a stochastic return $r \in [0, R]$, $E(r) = \bar{r} > i$;
 - (2) how to allocate these assets between a taxable account and a tax-sheltered account.

Interest on bonds held in the taxable account (TA) is taxed at rate τ ($0 < \tau < 1$), while equity returns are taxed at rate $\lambda\tau$ ($0 < \lambda < 1$). Assets placed in the tax-sheltered account (TSA) are tax-exempt. An amount up to $V < W$ may be placed in the tax-sheltered account.

- A. Derive the optimal portfolio, in terms of the amounts of debt and equity held in each account, for an individual who is risk neutral; perform the same exercise for an individual who is infinitely risk averse.
- B. Show that, regardless of an individual's degree of risk aversion, it will never be optimal for the individual to hold equity in the TSA and bonds in the TA at the same time. (Hint: starting with such an initial allocation, show that adjustments in the composition of the two accounts would permit the investor to achieve higher aggregate after-tax earnings on debt for a given distribution of aggregate after-tax earnings on equity.)
- C. Suppose that the tax-sheltered account conveys its tax benefit through deduction rather than through tax exemption. That is, suppose that initial contributions to the tax-sheltered account are deductible at rate τ and withdrawals in the subsequent period are taxable at the same rate, τ . How would this affect your answers to parts A and B?